$\ell=0$ **TO** $\ell=1$ **TRANSITION FORM FACTORS**

J. Bolz [1] [2] and P. Kroll [3]

Fachbereich Physik, Universität Wuppertal,
D-42097 Wuppertal, Germany

J. G. Körner [4]

Institut für Physik, Johannes-Gutenberg-Universität, Staudinger Weg 7,
D-55099 Mainz, Germany

Abstract

A method is proposed to extend the hard scattering picture of Brodsky and Lepage to transitions between hadrons with orbital angular momentum $\ell=0$ and $\ell=1$. The use of covariant spin wave functions turns out to be very helpful in formulating that method. As a first application we construct a light-cone wave function of the nucleon resonance $N^*(1535)$ in the quark-diquark picture. Using this wave function and the extended hard scattering picture, the N–N^* transition form factors are calculated at large momentum transfer and the results compared to experimental data. As a further application of our method we briefly discuss the π–a_1 form factors in an appendix.

PACS: 13.40.Fn; 12.38.Bx; 14.20.Gk; 14.40.Cs

1   Introduction

Exclusive processes at large momentum transfer are expected to belong to the few subjects of hadronic physics amenable to perturbative QCD. The "hard scattering picture" (HSP) [1] which was first formulated by Brodsky and Lepage offers a systematic framework for the calculation of hadronic quantities like form factors. The basic assumption of that model is that the physics at small momentum scales decouples from the hard processes and can be factorized into process independent distribution amplitudes which are specific to the hadrons involved. In order to accomplish this factorization the assumption of collinear constituents has been essential. That assumption is, however, only reasonable for ground state hadrons. For other hadrons, having orbital angular momentum $\ell \neq 0$, non-vanishing transverse momenta $(k_\perp)$ are mandatory. In this paper we attempt a generalization of the HSP such that also transitions between ground state hadrons and excited states can be treated. Therefore, the inclusion of transverse momenta is required which, besides allowing the investigation of $\ell = 0 \rightarrow \ell \neq 0$ has other consequences and applications. As was shown by Li and Sterman recently [2] the inclusion of intrinsic transverse momentum helps to formulate the HSP in a more self-consistent way, in particular it serves to overcome difficulties connected with the running of the strong coupling constant $\alpha_S(Q^2)$. It may also allow to study $\ell \neq 0$ admixtures to ground state hadrons as for instance the proton. Such admixtures generate hadronic helicity flip contributions and may therefore explain the experimentally observed violation of the helicity sum rule of the standard HSP [1]. Thus, one can expect that a consistent treatment of $k_\perp$-dependent contributions substantially enriches the HSP.

As we are going to demonstrate below the use of covariant spin wave functions is extremely helpful in formulating the generalization of the HSP. Covariant spin wave functions for hadrons with internal orbital angular momentum have first been used by Kühn et al. [3, 4]. These authors constructed spin wave functions via the ls coupling scheme and expanded both the wave function and the transition amplitude of a given process in terms of the relative momentum. These features are also present in the covariant formalism that we are going to introduce in Sect. 2.

In this paper the main interest is focussed on the calculation of the $N \rightarrow N^*(1535)$ transition form factors. In the literature there are several publications on this issue of which we like to discuss only a few. First of all Carlson and Poor [5] calculated the $N \rightarrow N^*(1535)$ transition form factors within the HSP. They disregarded the transverse momenta of the quarks and constructed a symmetric distribution amplitude for the N^* the lowest moments of which were constrained by QCD sum rules. In so far Carlson and Poor's distribution amplitude is similar to that of the nucleon, albeit with the opposite parity obtained by modifying the nucleon covariants by insertion of γ_5. In a similar manner the same process has been calculated in the quark-diquark picture [6]. Proceeding in such a manner one obtains the same expressions for the transition form factors as for the elastic nucleon form factors, up to kinematical factors. In our opinion such a treatment seems questionable because according to the $SU(6)$ quark model the N^* spin-flavour function should be of mixed symmetry and thus should differ markedly from that of the nucleon. Konen and Weber studied the $N \rightarrow N^*$

2

transition in a relativistic constituent quark model in the low Q^2 regime [7]. Although their model is of limited applicability at large Q^2 it contains a correct treatment of the N^* wave function in the 3-quark picture. For the nucleon wave function they used the light-cone model of Dziembowski and Weber [8] and for the N^* they constructed a proper $SU(6)$ wave function from a totally symmetric Gaussian momentum distribution function and negative-parity invariants with an explicit relative momentum dependence.

The goal of this paper is to reanalyze the $N \to N^*$ transition form factors within the HSP but using a proper N^* wave function now. We will assume the baryons, N and N^*, to consist of a quark and a diquark as in Ref. [6]. The diquark, being a cluster of two valence quarks and a certain amount of glue and sea quark pairs, is regarded as a quasi-elementary constituent, which partly survives medium hard collisions. The composite nature of the diquark is taken into account by diquark form factors which are parameterized in such a way that the pure quark picture of Brodsky and Lepage emerges asymptotically.

The diquark picture models non-perturbative effects, in fact correlations in the baryon wave function, which are known to play an important rôle at moderately large momentum transfer. The quark-diquark model of baryons has turned out to work rather well for exclusive reactions. With a common set of parameters specifying the diquarks and process independent wave functions for the involved hadrons a good description of a large number of exclusive reactions has been accomplished by now [9, 10, 11].

The paper is organized as follows: In Sect. 2 we lay down the foundation of how to treat transitions between $\ell = 0$ and $\ell = 1$ states in the hard scattering picture including a brief review of the $0 \to 0$ processes considered to date. Sect. 3 is devoted to the formalism of covariant spin wave functions which we will use for the description of quark-diquark bound states. The calculation of these form factors is described in Sect. 4 and the results are presented in Sect. 5. Finally, Sect. 6 is devoted to our summary. The appendices contain a description of how to construct covariant spin wave functions using the ls coupling scheme and an examplary calculation of the $\ell = 0$ to $\ell = 1$ meson transition $\pi \to a_1$.

2   The idea

We are going to calculate the $N(\frac{1}{2}^+) \to N^*(\frac{1}{2}^-)$ transition form factors at large momentum transfer. For this calculation we use the hard scattering picture as developed by Brodsky and Lepage [1], but generalized in such a way that also hadrons with non-zero orbital angular momentum between their constituents can be treated. The main idea of that generalization is presented in this section; its explicit application to the N–N^* transitions will be discussed in Section 5. The approach we present here may also be applied to other hadron-hadron transitions with $\ell \neq 0$.

As usual we use the light-cone approach, or — equivalently — infinite momentum frame techniques, to formulate the generalized hard scattering picture. In the light-cone approach, which is a covariant framework for describing a composite system with a fixed number of

constituents, a hadron is described by a momentum-space Fock basis. The coefficients of the various states of that basis represent wave functions defined at equal light-cone time $\tau = t + z$, rather than at equal t. Since the light-cone approach enables one to completely separate the kinematical and dynamical features of the Poincaré invariance [12, 13], there is a factorization of the basis hadronic states into the kinematical and dynamical parts, i.e. the overall motion of the hadron is decoupled from the internal motion of the constituents. Supppose the hadron's momentum is $P = (P^+, P^-, \vec{P}_\perp)$ where $P^+ = P^0 + P^3$ and $P^- = (P^0 - P^3)/2$. To accomplish the factorization into overall and internal motion, the momentum of the j-th parton belonging to a given Fock state is characterized by the fraction $x_j = p_j^+/P^+$ and by the transverse momentum $\vec{p}_{\perp j} = x_j \vec{P}_\perp + \vec{k}_{\perp j}$. Momentum conservation provides two constraints on the parton momenta of that Fock state

$$\sum_{j=1}^{n} x_j = 1 \,; \qquad \sum_{j=1}^{n} \vec{k}_{\perp j} = 0 \,. \tag{2.1}$$

It can be shown [1, 13, 14] that the variables x_j and $\vec{k}_{\perp j}$ are invariant under all kinematical Poincaré transformations, i.e. under boosts along and rotations around the 3-direction as well as under transverse boosts. Moreover — and this is the essential point — the light-cone wave function $\psi = \psi(x_j, \vec{k}_{\perp j})$ for that Fock state is independent of the hadron's momentum and is invariant under these kinematical transformations too. Hence, ψ is determined if it is known at rest. An ordinary equal-time wave function does not possess these properties. The wave function ψ may depend on internal quantum numbers such as helicities, a possibility which we omit for the present discussion. We emphasize that only the spin operator J_3 is purely kinematical, whereas the other two spin operators depend on the interaction. Consequently, rotational invariance is difficult to implement in the light-cone formalism. As long as only hadrons with zero orbital angular momenta between their constituents are considered, this difficulty disappears [1]. This is also the case for the form factors of transitions between $\ell = 0$ and $\ell = 1$ hadrons as we are going to show below. In general, however, this is not true. Thus, for instance, for exclusive decays of the P-wave charmonia a consistent, rotational invariant light-cone approach has not yet been achieved. In order to preserve rotational invariance one usually describes the charmonium state by an ordinary equal-t wave function, whereas light-cone wave functions are used for the final state hadron [15].

It can be shown [1] that the contributions from the valence Fock state dominate at large momentum transfer, higher Fock state contributions are suppressed by powers of α_S/Q^2. Therefore, one customarily assumes valence Fock state dominance in applications of the hard scattering picture, although, at moderately large momentum transfer — the region where data is available — it is by no means obvious that contributions from higher Fock states are indeed negligible [16, 17]. The valence Fock state dominance is particularly questionable for $\ell \neq 0$-hadrons [18]. Nevertheless, we will make use of this assumption.

Let us now discuss the main features of the $N - N^*$ transition form factor calculation at large or moderately large momentum transfer. Because of the reasons mentioned in the introduction we regard a baryon as a bound state of a quark and a diquark where the latter is treated as a quasi-elementary constituent (for details, see Sects. 3 and 5). We perform the calculation in a frame in which both baryons move along the 3-direction. In particular

4

we use the opposite momentum brick wall frame defined by $\hat{P}_i^\mu = (E_i - P, (E_i + P)/2, \vec{0}_\perp)$ and $\hat{P}_f^\mu = (E_f + P, (E_f - P)/2, \vec{0}_\perp)$ where $\hat{P}_j^\mu$ and $\hat{P}_f^\mu$ denote the momenta of the initial and final state baryon, respectively. Any other frame that is obtained by a boost along the 3-direction from the brick wall frame belongs to the set of frames in the 3-direction, hence the infinite momentum frame too.

In order to bring out the main features for the moment we ignore all complications in connection with flavour wave functions as well as the possibility that several different quark-diquark configurations may contribute to a given baryon. We make the following ansatz for a spin $\frac{1}{2}$-baryon of type a [1]

$$| a; \vec{P}_a, \lambda_a \rangle = \int \frac{dx_1 \, d^2 k_\perp}{16\pi^3} \, \psi_a(x_1, \vec{k}_\perp) \, \Gamma_a \, u_a(\vec{P}_a, \lambda_a) , \tag{2.2}$$

where P_a and λ_a are the momentum and the helicity of the baryon; u_a represents its light-cone spinor. It is normalized as $\bar{u}u = 1$. The variables x_1 and $\vec{k}_\perp$ refer to the quark; the diquark variables are $x_2 = 1 - x_1$ and $-\vec{k}_\perp$. Γ_a represents the covariant spin wave function of the baryon [19, 20]. Here, in this section, we only need a few basic features of the spin wave functions; their explicit forms for the baryons under consideration will be specified in Section 3. A spin wave function depends on the hadronic spin and parity as well as on the quantum numbers of the constituents. But, with the exception of $k_\perp$, it does not depend on the momenta or polarisation vectors (helicities) of the constituents. This fact entails an enormous technical advantage in the calculation of matrix elements. One of the basic ingredients of the hard scattering picture is the so-called collinear approximation which says that all constituents move along the same direction as their parent hadron up to a scale of the order of the Fermi momentum $< k_\perp^2 >^{1/2}$ which is typically a few 100 MeV. This collinear approximation justifies an expansion of the spin wave function in terms of a power series in $\vec{k}_\perp$ or, in order to retain the covariant formulation, in $K^\mu = (K^+ = 0, K^- = 0, \vec{k}_\perp)$. Up to terms linear in K^μ this expansion reads

$$\Gamma_a \left(\ell = 0\right) = \Gamma_{a0}(K = 0) + \Delta\Gamma_{a0}^\alpha(K = 0) \, K_\alpha + \mathcal{O}(k_\perp^2) , \tag{2.3}$$

for a $\ell = 0$ baryon. The light-cone wave function ψ_a of the baryon a is a scalar function in this case; it depends only on $k_\perp^2$, not on $\vec{k}_\perp$.

Contrary to the above ansatz, the momentum $K^\mu = (p_1^\mu - p_2^\mu)/2$ (see Sect. 3) possesses a non-vanishing $K^- (= (K^0 - K^3)/2)$-component in general. Writing the constituent momenta in a covariant fashion as $(j = 1, 2)$

$$p_j^\mu = x_j P^\mu + k_j^\mu , \tag{2.4}$$

where, in agreement with the required invariance under the kinematical Poincaré transformations,

$$k_1^\mu = (0, k_1^-, \vec{k}_\perp) , \qquad k_2^\mu = (0, k_2^-, -\vec{k}_\perp) . \tag{2.5}$$

---

[1] Note that, for conciseness, we have omitted colour indices and a factor representing the plane waves.

For the minus-component of the relative momentum we have

$$K^- = \frac{1}{2}(k_1^- - k_2^-)\,. \tag{2.6}$$

As is customary in the parton model, we neglect the binding energy and consider the constituents as on-shell particles. That possibly crude approximation can be achieved by putting the individual k_j^--components to zero. With this choice also K^- is zero and hence $P{\cdot}K = 0$. As we will see this choice leads to a very elegant and compact representation of the transition matrix elements. Under these circumstances the constituent velocities equal that of their parent hadron up to corrections of order $k_\perp$.

In the case of non-zero orbital angular momentum between the two constituents of a given baryon a non-zero transverse momentum is required. Indeed one can easily show [19] that in the case of $\ell = 1$ the following expansion holds for the spin wave function

$$\Gamma_a(\ell{=}1) = \Gamma_{a1}^\alpha(K{=}0)\,K_\alpha + \Delta\Gamma_{a1}^{\alpha\beta}(K{=}0)\,K_\alpha K_\beta + \mathcal{O}(k_\perp^3)\,. \tag{2.7}$$

ψ_a in (2.2) now represents a reduced wave function, i.e. the full wave function with a factor K^μ removed from it. In the non-relativistic case (see Appendix A) the factor K^μ arises from the spherical function and ψ_a is related to the radial function. As in the $\ell = 0$ case the reduced wave function ψ_a depends only on $\vec{k}_\perp^2$.

In both cases, $\ell{=}0$ and $\ell{=}1$, we normalize the spin wave functions in such a way that

$$\bar{u}_a(P_a,\lambda_a')\,\bar{\Gamma}_a\,\Gamma_a\,u_a(P_a,\lambda_a) = k_\perp^{2\ell}\,\delta_{\lambda_a\lambda_a'} + \mathcal{O}(k_\perp^{2\ell+2})\,, \tag{2.8}$$

where, as usual, $\bar{\Gamma}_a = \gamma_0\,\Gamma_a^\dagger\,\gamma_0$. Proper state normalization requires the condition

$$\int \frac{dx_1\,d^2k_\perp}{16\pi^3}\,|\,k_\perp^\ell\,\psi_a(x_1,k_\perp)\,|^2 = \kappa_a \leq 1\,. \tag{2.9}$$

κ_a is to be interpreted as the probability of finding the given quark-diquark configuration inside the baryon a.

Within the hard scattering picture the current matrix elements for the transitions from a baryon i to a baryon f are represented by convolutions of the corresponding wave functions and hard scattering amplitudes T_H. The latter are to be calculated perturbatively from an appropriate set of Feynman diagrams:

$$\langle\, f;\,\vec{P}_f,\,\lambda_f \mid J^\mu \mid i;\,\vec{P}_i,\,\lambda_i\,\rangle = \bar{u}_f(P_f,\lambda_f)$$
$$\times \int \frac{dx_1\,d^2k_\perp\,dy_1\,d^2k_\perp'}{(16\pi^3)^2}\,\psi_f^*(y_1,k_\perp')\,\bar{\Gamma}_f\,T_H^\mu(x_1,y_1,Q,K,K')\,\Gamma_i\,\psi_i(x_1,k_\perp)\,u_i(P_i,\lambda_i)\,. \tag{2.10}$$

$Q^2\,(\geq 0)$ is the usual invariant momentum transfer from the initial to the final state baryon. For the final baryon the momentum fractions, the transverse momentum and the relative momentum are denoted by y_j, $k_\perp'$ and K' respectively. Comparison of (2.10) with a standard decomposition of the current matrix elements in terms of covariants and invariant form factors leads to a determination of the form factors within the hard scattering model.

6

At large Q^2 the intrinsic transverse momentum dependence of the hard scattering amplitude provides only a higher twist correction. In view of this, we also expand the amplitude in a power series of the transverse momenta:

$$T_H^\mu(x_1, y_1, Q, K, K') =$$
$$T_H^\mu(x_1, y_1, Q, K=K'=0) + \left.\frac{\partial T_H^\mu}{\partial K^\alpha}\right|_{K=K'=0} K^\alpha + \left.\frac{\partial T_H^\mu}{\partial K'^\alpha}\right|_{K=K'=0} K'^\alpha + \mathcal{O}(k_\perp^2, k_\perp'^2). \quad (2.11)$$

Let us now demonstrate that on keeping only the first term on the r.h.s. of (2.11) one recovers the usual $i(\ell=0) \to f(\ell'=0)$ hard scattering formula. Inserting (2.11) and (2.3) or (2.7) into (2.10) and noting that

$$\int \frac{dx_1 \, d^2 k_\perp^{(')}}{16\pi^3} \, K^{(')\mu} \, \psi_{i(f)}(x_1, k_\perp) = 0, \quad (2.12)$$

we find

$$\langle f(\ell'=0); \vec{P}_f, \lambda_f \mid J^\mu \mid i(\ell=0); \vec{P}_i, \lambda_i \rangle = \bar{u}_f(P_f, \lambda_f) \int \frac{dx_1 d^2 k_\perp dy_1 d^2 k_\perp'}{(16\pi^3)^2} \psi_f^*(y_1, k_\perp')$$
$$\times \quad \bar{\Gamma}_{f0}(K'=0) \, T_H^\mu(x_1, y_1, Q, K=K'=0) \, \Gamma_{i0}(K=0) \, \psi_i(x_1, k_\perp) \, u_i(P_i, \lambda_i)$$
$$+ \quad \mathcal{O}(k_\perp^2, k_\perp'^2). \quad (2.13)$$

The integrations over the transverse momenta apply only to the wave functions. Carrying out these integrations formally, one arrives at the so-called distribution amplitudes (DA) $(a = i, f)$.

$$f_a \, \phi_a(x_1) = \int \frac{d^2 k_\perp}{16\pi^3} \, \psi_a(x_1, k_\perp), \quad (2.14)$$

where, by convention, the distribution amplitudes are defined in such a way that

$$\int dx_1 \, \phi_a(x_1) = 1. \quad (2.15)$$

We have factored out the coupling factor f_a which represents the configuration space wave function at the origin. With these defintions, Eq. (2.13) simplifies to

$$\langle f(\ell'=0); \vec{P}_f, \lambda_f \mid J^\mu \mid i(\ell=0); \vec{P}_i, \lambda_i \rangle =$$
$$\bar{u}_f(P_f, \lambda_f) \int dx_1 dy_1 \, f_f \, \phi_f^*(y_1) \, \bar{\Gamma}_{f0}(K'=0) \, T_H^\mu(x_1, y_1, Q, K=K'=0)$$
$$\times \quad \Gamma_{i0}(K=0) \, f_i \, \phi_i(x_1) \, u_i(P_i, \lambda_i) + \mathcal{O}(k_\perp^2, k_\perp'^2). \quad (2.16)$$

This is the standard hard scattering formula as derived by Brodsky and Lepage [1]. The integrand factorizes into long-distance physics, contained in the distribution amplitudes, and short-distance physics, contained in the hard scattering amplitudes. In principle the distribution amplitudes depend logarithmically on Q as a consequence of the QCD evolution

[1] which will, however, not be taken into account in our calculation.
For the case $i(\ell=0) \to f(\ell'=1)$, on the other hand, we find

$$\langle f(\ell'=1); \vec{P}_f, \lambda_f \mid J^\mu \mid i(\ell=0); \vec{P}_i, \lambda_i \rangle = \bar{u}_f(P_f, \lambda_f) \int \frac{dx_1 d^2 k_\perp dy_1 d^2 k'_\perp}{(16\pi^3)^2} \psi_f^*(y_1, k'_\perp)$$

$$\times \quad K'_\alpha K'_\beta \left[\bar{\Gamma}^\alpha_{f1}(K'=0) \frac{\partial T^\mu_H}{\partial K'_\beta}\bigg|_{K=K'=0} + \Delta\bar{\Gamma}^{\alpha\beta}_{f1}(K'=0) T^\mu_H(K=K'=0) \right]$$

$$\times \quad \Gamma_{i0}(K=0) \psi_i(x_1, k_\perp) u_i(P_i, \lambda_i) + \mathcal{O}(k_\perp^2 k'^2_\perp, k'^4_\perp), \tag{2.17}$$

to the requisite order of $k_\perp$. Note that terms linear in K_α or K'_α drop out after integration. One notes that the first order correction term to the $\ell'=1$-spin wave function enters the matrix element to the same order as the leading wave function term. Thus, in contrast to the $\ell = 0$-case, the correction term to the $\ell' = 1$-spin wave function is required at leading order. [2] Again, as in the $\ell=0$-case, the transverse momentum integrations apply only to the wave functions. With the aid of the covariant integration formula

$$\int \frac{d^2 k'_\perp}{16\pi^3} K'_\alpha K'_\beta \psi_f(y_1, k'_\perp) = f_f \phi_f(y_1) I_{\alpha\beta}/2, \tag{2.18}$$

the tensor integration in (2.18) can be done explicitly with the result [3]

$$I_{\alpha\beta} = -g_{\alpha\beta} + \frac{1}{M_f^2}(P_{f\alpha}P_{f\beta} - \bar{P}_{f\alpha}\bar{P}_{f\beta}). \tag{2.19}$$

The momentum $\bar{P}_{f\alpha}$ is given by ($P_f^+, -P_f^-, \vec{0}_\perp$) in the frame we are working. The remaining scalar integral serves to define a distribution amplitude in analogy to the $\ell = 0$ case, as mentioned before. One has

$$f_f \phi_a(y_1) = \int \frac{d^2 k'_\perp}{16\pi^3} k'^2_\perp \psi_f(y_1, k'_\perp), \tag{2.20}$$

where

$$\int dy_1 \phi_f(y_1) = 1. \tag{2.21}$$

Again we have factored out a coupling factor f_f which is to be interpreted as the derivative of the configuration space wave function with respect to r at the origin.

---

[2]A process for which this term is of particular relevance is the two-photon decay of the χ_0. The term $\sim \Gamma_1$ is zero. Hence, the familiar result for the decay width [21] is solely obtained from the term $\sim \Delta\Gamma_1$.

[3]The metric tensor reads in light-cone coordinates

$$g_{\alpha\beta} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}.$$

The scalar product of two 4-vectors is $a \cdot b = a^+ b^- + a^- b^+ - \vec{a} \cdot \vec{b}$.

Inserting (2.18-2.21) into (2.17) we obtain

$$
\begin{aligned}
\langle\, f(\ell'\!=\!1);\ \vec{P}_f,\ \lambda_f \,|\, J^\mu \,|\, i(\ell\!=\!0);\ \vec{P}_i,\ \lambda_i\,\rangle \;&=\; \bar{u}_f(P_f,\lambda_f)\int dx_1 dy_1\\
\times\ \ \frac{1}{2}f_f\,\phi_f^*(y_1)\,I_{\alpha\beta}&\left[\bar{\Gamma}_{f1}^\alpha(K'\!=\!0)\left.\frac{\partial T_H^\mu}{\partial K'_\beta}\right|_{K=K'=0}\;+\;\Delta\bar{\Gamma}_{f1}^{\alpha\beta}(K'\!=\!0)\,T_H^\mu(K\!=\!K'\!=\!0)\right]\\
\times\ \ \Gamma_{i0}(K\!=\!0)\,f_i\,\phi_i(x_1)\,u_i(P_i,\lambda_i)\;&+\;\mathcal{O}(k_\perp^2 k_\perp'^2, k_\perp'^4)\,.
\end{aligned}
\tag{2.22}
$$

This is the generalized hard scattering formula. We will use it to calculate the $N \to N^*$-transition form factors (see Sect. 4).
The following remarks are in order:

i) Eq. (2.22) factorizes into long and short distance physics. As in the standard hard scattering formula (2.16) the long distance physics is contained in the distribution amplitudes, the short distance physics in the hard scattering amplitude and its derivative.

ii) The generalization of (2.22) to $\ell \to \ell'$-transitions for any values of ℓ and ℓ' is straightforward.

iii) We have presented the generalized hard scattering formula for baryonic transitions. The same approach, however, also holds for mesonic transitions. One only has to replace the baryon spinors in (2.22) by a trace over Dirac matrices. In Appendix B we demonstrate this for the $\pi \to a_1$ transitions.

iv) We have used the quark-diquark model for the baryons because of our actual phenomenological interest. The approach to $\ell \to \ell'$-transitions that we presented in this section can easily be adapted to the pure quark picture of baryons.

v) Recently, Sterman and Li [2] have modified the hard scattering picture (for $\ell\!=\!0 \to \ell\!=\!0$ transitions). They kept the transverse momentum dependence of the hard scattering amplitude and took into account radiative (Sudakov) corrections. Because of that the perturbative QCD contribution to form factors becomes self-consistent even for momentum transfers as low as a few GeV. Here self-consistency means that most of the result comes from hard regions where α_S, the strong coupling constant, is small. As has been pointed out in [17] the consistency of the entire approach also requires the inclusion of the intrinsic transverse momentum dependence of the hadronic wave function. The approach put forward by Sterman and Li challenges previous objections [16, 22] against the use of the hard scattering picture. Our approach to $\ell \to \ell'$-transitions can be generalized such as to include Sudakov corrections as well. However, we dispense with these corrections in this explorative attempt to calculate $\ell\!=\!0 \to \ell\!=\!1$ transitions.

3 The covariant spin wave functions for the nucleon and the S_{11}-nucleon resonance

We now discuss in some detail the wave functions for the two baryons of interest. As we have mentioned repeatedly we consider baryons to be bound states of quarks and diquarks. We emphasize that for each baryon under consideration the spin-flavour wave functions are constructed in such a way that after resolving the diquarks into two quarks the corresponding representation of the static $SU(6)$-model emerges. This requirement puts constraints on possible quark-diquark configurations inside a baryon as well as on relative factors between the different terms in the quark-diquark wave function.

Covariant baryon spin wave functions have been derived in a very general way in Ref. [19]. They are presented in that paper in a way such that the spin wave functions can easily be interpreted as originating from a quark-diquark picture although in Ref. [19] this picture itself has not been used.

The introduction of covariant spin wave functions allows one to covariantly calculate the probability amplitude that a given spin configuration of the quark and diquark constituents is contained in a given baryon of any spin and parity. In this sense the covariant spin wave functions represent the Clebsch-Gordan coefficients of a particular rest frame spin coupling scheme boosted to the moving frame and can be derived from the standard non-relativistic ls-coupling scheme (see Appendix A).

In the rest frame of the baryon, where the velocity 4-vector takes the form $v_a^\mu = P_a^\mu/M_a = (1, \vec{0})$, the covariant spin coupling reduces to couplings of spin objects transforming under the three dimensional rotation group $O(3)$ or $SU(2)$. In order to achieve this, one has to assure that the time or spin zero components of Lorentz tensors that are used in the construction of the covariant spin wave functions decouple in the rest frame. One therefore has to use 4-transverse angular momentum vectors and tensors in the construction of the covariant spin wave functions. For example when K^μ is the relative momentum

$$K^\mu = (p_1^\mu - p_2^\mu)/2 \,, \tag{3.1}$$

each unit of orbital momentum will be represented by

$$K_{\perp a}^\mu = K^\mu - v_a \cdot K \, v_a^\mu \,. \tag{3.2}$$

In the rest frame clearly $K_\perp^\mu \to (0, \vec{k})$ and one has the appropriate object transforming as a 3-vector under $O(3)$. Considering the fact that the spin wave functions act on baryon spinors one can easily convince oneself that the relative momentum is only determined up to a multiple of the hadron momentum. Combining this freedom with the zero binding energy approximation, one can define K as we did in Sect. 2, namely in such a way that it has only the transverse components $\vec{k}_\perp$. This choice, although not forced, is very convenient.

Similarly, for a baryon of type a, one has to work with the 4-transverse Dirac matrix

$$\gamma_{\perp a}^\mu = \gamma^\mu - v_a^\mu \, \slashed{v}_a = (g^{\mu\nu} - v_a^\mu v_a^\nu) \, \gamma_\nu \,, \tag{3.3}$$

which clearly reduces to $(0, \vec{\gamma})$ in the rest frame of the baryon representing a spin operator with the correct transformation property under $O(3)$. We mention that only one of the off-diagonal 2×2 matrices in the rest frame form of $\gamma^{\mu}_{\perp a}$ is of relevance since we are working with fermions (and not anti-fermions).

In writing down the baryon states we have to consider the possibility that a baryon may be composed of several quark-diquark configurations. The ground state nucleon is made up of two configurations when admixtures from configurations with $\ell \neq 0$ are neglected. In terms of the non-relativistic quark-model this means that we assume the nucleon to be a pure state of the $SU(6)$ $\{56\}$ multiplet. The two configurations of the nucleon consist of a quark and either a spin 0, isospin 0 diquark (S) or a spin 1, isospin 1 diquark (V). According to Sect. 2, in particular Eqs. (2.2, 2.3, 2.14), we therefore write a nucleon state as

$$| N; \vec{P}_i, \lambda_i \rangle \; = \; \int dx_1 \, [\, f_{NS} \, \phi_{NS}(x_1) \, \chi_{NS} \, \Gamma_{NS} \; + \; f_{NV} \, \phi_{NV}(x_1) \, \chi_{NV} \, \Gamma^{\mu}_{NV} \,] \; u_N(P_i, \lambda_i) \,, \quad (3.4)$$

where the covariant spin wave functions are given by

$$\Gamma_{NS} \; = \; 1 \,; \qquad \Gamma^{\mu}_{NV} \; = \; \frac{1}{\sqrt{3}} \, \gamma^{\mu}_{\perp i} \, \gamma_5 \; = \; \frac{1}{\sqrt{3}} \, (\gamma^{\mu} + v^{\mu}_i) \, \gamma_5 \,. \qquad (3.5)$$

In (3.5) we have made use of the fact that the spin wave function acts on the baryon spinor, i.e. we have replaced $\not{v}_i$ by 1. The spin wave functions are normalized according to (2.8). We do not need K-dependent correction terms to these spin wave functions to the order we are working (see (2.13)). The χ's in Eq. (3.4) represent flavour functions, which, for the proton and neutron, take the form

$$\begin{aligned} \chi_{pS} &= u \, D^0_{[u,d]} \,, & \chi_{pV} &= [u \, D^1_{\{u,d\}} - \sqrt{2} \, d \, D^1_{\{u,u\}}]/\sqrt{3} \,, \\ \chi_{nS} &= d \, D^0_{[u,d]} \,, & \chi_{nV} &= -[d \, D^1_{\{u,d\}} - \sqrt{2} \, u \, D^1_{\{d,d\}}]/\sqrt{3} \,, \end{aligned} \qquad (3.6)$$

where $D^0_{[u,d]}(D^1_{\{u,d\}})$ stands for an isospin 0 (1) diquark made of a u and a d quark.

The form (3.4) of the nucleon state has been used in many applications of the hard scattering picture. Thus, for instance, the electromagnetic form factors of the nucleon have been investigated in both the space-like and the time-like regions [9]. Also Compton-scattering, two photon annihilation into proton-antiproton, photoproduction of mesons as well as some exclusive decay processes have been studied. For a recent review of applications of the quark-diquark picture, see [10]. All of these studies have been carried out with the following distribution amplitudes

$$\phi_{NS}(x_1) \; = \; \mathcal{C}_{NS} x_1 x_2^3 \exp\left[-\beta_N^2 (m_q^2/x_1 + m_S^2/x_2)\right] \,, \qquad (3.7)$$

$$\phi_{NV}(x_1) \; = \; \mathcal{C}_{NV} x_1 x_2^3 (1 + 5.8 x_1 - 12.5 x_1^2) \exp\left[-\beta_N^2 (m_q^2/x_1 + m_V^2/x_2)\right] \,. \qquad (3.8)$$

We reiterate that x_1 and $x_2 = 1 - x_1$ refer to the longitudinal fractions of the quark and the diquark, respectively. The DAs represent a kind of harmonic oscillator wave function transformed to the light-cone. The masses in the exponentials are, therefore, constituent masses since they enter through a rest frame wave function. For the quark mass we take 330

11

MeV, whereas for the diquark masses a value of 580 MeV is used. The oscillator parameter β_N is taken to be 0.5 GeV^{-1}. This value is obtained by the requirement that the root mean square (rms) transverse momentum $< k_\perp^2 >^{1/2}$ is 600 MeV (as found e.g. by the EMC [23] in a study of transverse momentum distributions in semi-inclusive deep inelastic μ–p scattering), assuming a Gaussian $k_\perp$ dependence for the full wave function

$$\sim \ \exp\left[-\beta_N^2 \ \frac{k_\perp^2}{x_1 x_2}\right] \ . \tag{3.9}$$

The exact values of the masses and that of the oscillator parameter are not very important since the exponentials in (3.7, 3.8) are only of importance in the end-point regions. The constants $\mathcal{C}_{NS}$ and $\mathcal{C}_{NV}$ in (3.7, 3.8) are fixed by the condition (2.15) ($\mathcal{C}_{NS} = 25.96$; $\mathcal{C}_{NV} = 22.28$). The more complex form of the distribution amplitude for the quark-vector diquark configuration implies a smaller mean value of x_1 than obtained with the distribution amplitude for the quark-scalar diquark configuration. In other words, on the average the V-diquark carries a larger fraction of the proton's momentum than the S-diquark. This feature is in accord with the current expectation that the V-diquark mass is larger than that of the S-diquark.

The constants f_{NS} and f_{NV}, the values of the two configuration space wave functions at the origin, have also been fixed in the previous applications of the quark-diquark models. For these constants the values $f_{NS} = 73.85$ MeV and $f_{NV} = 127.7$ MeV have been obtained. For the calculation of the N–N^* transition form factors we use the distribution amplitudes (3.7) and (3.8) with the quoted values of the various parameters.

For the N^* the situation is much more complex. On the one hand the same two types of diquarks, S and V contribute to a N^* state. This requires one unit of orbital angular momentum between quark and diquark. For these two configurations the appropriate flavour functions are identical to those of the nucleon (Eq. (3.6)). The zeroth order terms of the p-wave covariant spin wave functions read [4]

$$\Gamma_{N^*S}^\alpha \ = \ \gamma^\alpha \, \gamma_5 \,; \qquad \Gamma_{N^*V}^{\mu\alpha} \ = \ -\frac{1}{\sqrt{3}} \, \gamma_{\perp f}^\mu \, \gamma^\alpha \,; \tag{3.10}$$

In terms of the non-relativistic quark model these configurations correspond to mixed-symmetric states. As in the case of $\ell = 0$ states discussed in Sect. 2 we also need correction terms $\sim K^\beta$ to the spin wave functions (3.10). As described in Appendix A we obtain these corrections from expanding the quark spinors and the diquark polarisation vectors relative to the direction of the N^*. Quarks and diquarks are considered as free, on-shell particles.

---

[4]We note that $\Gamma_{N^*V}^{\mu\alpha}$ corresponds to the following case in the ls coupling scheme: The spins of quark and vector diquark are coupled in a state of total spin 1/2; in the second step the total spin is coupled with the orbital angular momentum. The other possibility, namely total spin 3/2, leads to the $S_{11}(1700)$ nucleon resonance. The corresponding spin wave function in this case reads $\frac{1}{\sqrt{6}}\left[g^{\mu\alpha} + \gamma_{\perp f}^\mu\gamma^\alpha\right]$. Note that the state corresponding to $\Gamma_{N^*V}^{\mu\alpha}$ is a linear superposition of the two $J^P = 1/2^-$ states relevant for p-wave heavy baryon states [24].

We find

$$\Delta\Gamma^{\alpha\beta}_{N^*S} = \frac{1}{2y_1 M_f}\,\gamma_5\,g^{\alpha\beta}$$

$$\Delta\Gamma^{\mu\alpha\beta}_{N^*V} = -\frac{1}{\sqrt{3}}\frac{1}{2y_1 M_f}\,[2g^{\mu\alpha} - (\gamma^{\mu}_{\perp f} - 2\frac{y_1 P^{\mu}_f}{y_2 M_f})\gamma^{\alpha}]\,\gamma^{\beta}\,. \qquad (3.11)$$

The Lorentz indices α and β are to be contracted with K^α and K^β (see (2.7)). We stress that the above correction terms are model dependent contrary to the zeroth order spin wave functions.

On the other hand the orbital angular momentum that is required to generate the parity of the N^* may also be inside the diquark. Hence, in this case, there is no orbital angular momentum between quark and diquark, i.e. we are back to the $\ell=0$ case. In order to account for that possibility we have to introduce three new diquarks which are parity partners of the S- and V-diquarks, i.e. a spin 0, isospin 0 diquark (P), a spin 1, isospin 0 diquark (A_0) and a spin 1, isospin 1 diquark (A_1). In terms of the non-relativistic quark model the qP-, qA_0- and qA_1-configurations correspond to mixed antisymmetric states. As will be discussed in Sect. 4 two of these configurations, namely qP and qA_1, do not contribute to N–N^* transitions or can be neglected. Therefore, we write a $N^*(1535)$ state as

$$\begin{aligned}
|\,N^*;\,\vec{P}_f,\,\lambda_f\,\rangle\,=\,&\int dy_1\,[\,f_{N^*S}\,\phi_{N^*S}(y_1)\,\chi_{N^*S}\,I_{\alpha\beta}/2\,(\Gamma^{\alpha}_{N^*S}\,+\,\Delta\Gamma^{\alpha\beta}_{N^*S})\\
&-\,f_{N^*V}\,\phi_{N^*V}(y_1)\,\chi_{N^*V}\,I_{\alpha\beta}/2\,(\Gamma^{\mu\alpha}_{N^*V}\,+\,\Delta\Gamma^{\mu\alpha\beta}_{N^*V})\\
&+\,f_{N^*A_0}\,\phi_{N^*A_0}(y_1)\,\chi_{N^*A_0}\,\Gamma^{\mu}_{N^*A_0}\,]\,u_{N^*}(P_f,\lambda_f)\,, \qquad (3.12)
\end{aligned}$$

where we have made use of the definitions (2.18)-(2.21). Obviously, $\chi_{N^*A_0}\,=\,\chi_{NS}$ and $\Gamma_{N^*A_0}\,=\,\Gamma_{NV}$.

In the ansatz (3.12) there are three new, a priori unknown distribution amplitudes and three constants f_{N^*j}. To further simplify the model and to reduce the number of free parameters, we assume that the N^* distribution amplitudes are of the same form as the corresponding nucleon distribution amplitudes, (3.7) and (3.8), for configurations containing a diquark of given spin. The three constants f_{N^*j} are considered as free parameters to be determined by fits to experimental data.

For the N^* there are two different oscillator parameters, $\beta_{N^*_0}$ and $\beta_{N^*_1}$, where the indices 0 and 1 corresponding to the two possible values of the orbital angular momentum ℓ between quark and diquark. For the qA_0-configuration ($\ell = 0$) we assume a $k_\perp$-dependence of the full wave function as in Eq. (3.9), and for N^*-configurations with a S- or V-diquark we assume a $k_\perp$-dependence as

$$\sim\,k_\perp\,\exp\left[-\beta^2_{N^*_1}\frac{k^2_\perp}{x_1 x_2}\right]\,, \qquad (3.13)$$

The ansatz (3.13) takes care of the fact that we deal with a $\ell=1$ system. Requiring again a value of 600 MeV for the rms transverse momentum, we find a value of 0.71 GeV^{-1} for the $\ell=1$ N^* oscillator parameter $\beta_{N^*_1}$. The corresponding normalization constants of the distribution amplitudes are $\mathcal{C}_{N^*S}\,=\,33.87$ and $\mathcal{C}_{N^*V}\,=\,28.65$. For the $\ell=0$ N^* oscillator

parameter $\beta_{N_0^*}$ we have 0.48 GeV^{-1} and the normalization constant $\mathcal{C}_{N^* A_0}$ is 31.97. For the constituent masses appearing in the distribution amplitudes (see Eqs. (3.7) and (3.8)) we take 330 MeV for the quark and 580 MeV for the S- and V-diquark as in the nucleon case and 1260 MeV for the A_0-diquark. The latter mass should be similar to that of the a_1 meson.

4   The calculation of the $N{-}N^*$ transition form factors

We denote the momentum, the helicity and the mass of the nucleon by P_i, λ_i and M_i and the corresponding quantities for the S_{11}-resonance by P_f, λ_f and M_f (see Fig. 1). The photon momentum is

$$q \;=\; P_f \,-\, P_i\,. \tag{4.1}$$

The $N{-}N^*$ transition matrix element is decomposed as [25]

$$\langle\, N^*;\, \vec{P}_f,\, \lambda_f \,|\, J^\mu \,|\, N;\, \vec{P}_i,\, \lambda_i \,\rangle \;=\; e_0\, \bar{u}_{N^*} \left[\, h_+(Q^2)/Q_-\, \mathcal{H}_+^\mu \,+\, h_0(Q^2)/Q_-\, \mathcal{H}_0^\mu \,\right] u_N\,, \tag{4.2}$$

where the helicity covariants are defined by

$$\mathcal{H}_+^\mu \;=\; i\, \slashed{P}_f\, \epsilon^{\mu\nu\rho\sigma}\, q_\nu\, P_{i\rho}\, \gamma_\sigma\,, \qquad\qquad \mathcal{H}_0^\mu \;=\; (P_i{\cdot}q\, q^\mu \,-\, q^2\, P_i^\mu)\, \gamma_5\,. \tag{4.3}$$

We have factored out the pseudothreshold factor

$$Q_- \;=\; (M_f \,-\, M_i)^2 \,+\, Q^2\,, \tag{4.4}$$

in order to avoid kinematical singularities in the helicity form factors at the pseudothreshold $(Q_- = 0)$.

If ϵ^μ is the polarisation vector of the virtual photon, then it can easily be seen from (4.2), using for instances the brick wall frame, that

$$\epsilon_\mu(\pm 1)\, \mathcal{H}_0^\mu \;=\; \epsilon_\mu(0)\, \mathcal{H}_+^\mu \;=\; 0\,. \tag{4.5}$$

Hence, the helicity form factor h_+ refers to transverse and and the form factor h_0 to longitudinal $N{-}N^*$ transitions. While the covariants (4.3) are manifestly gauge invariant, they are not free of kinematical constraints. At the pseudothreshold one has the following relation between the helicity form factors

$$h_+(\, Q^2 = -(M_f - M_i)^2\,) \;=\; \frac{M_i - M_f}{M_f}\, h_0\,(\, Q^2 = -(M_f - M_i)^2\,)\,. \tag{4.6}$$

In our variant of the hard scattering picture the $N{-}N^*$ transition matrix elements are given by

$$\langle\, N^*;\, \vec{P}_f,\, \lambda_f \,|\, J^\mu \,|\, N;\, \vec{P}_i,\, \lambda_i \,\rangle \;=\; \bar{u}_{N^*}(P_f,\, \lambda_f) \int dx_1 dy_1 \Big\{$$

14

$$\left[f_{N^*S}\,\phi_{N^*S}^*(y_1)\,\chi_{N^*S}^*\,\frac{I_{\alpha\beta}}{2}\left(\bar{\Gamma}_{N^*S}^\alpha\,\frac{\partial T_{HSS}^\mu}{\partial K_\beta'}\bigg|_{K'=0} + \Delta\bar{\Gamma}_{N^*S}^{\alpha\beta}\,T_{HSS}^\mu\right)\right.$$

$$+ f_{N^*A_0}\,\phi_{N^*A_0}^*(y_1)\,\chi_{N^*A_0}^*\,\bar{\Gamma}_{N^*A_0}^\rho\,T_{HSA_0\rho}^\mu \bigg]\,\chi_{NS}\,\Gamma_{NS}\,f_{NS}\,\phi_{NS}(x_1)$$

$$+ f_{N^*V}\phi_{N^*V}^*(y_1)\chi_{N^*V}^*\frac{I_{\alpha\beta}}{2}\left(\bar{\Gamma}_{N^*V}^{\alpha\rho}\,\frac{\partial T_{HVV\rho\sigma}^\mu}{\partial K_\beta'}\bigg|_{K'=0} + \Delta\bar{\Gamma}_{N^*V}^{\alpha\beta\rho}\,T_{HVV\rho\sigma}^\mu\right)\chi_{NV}\Gamma_{NV}^\sigma f_{NV}\,\phi_{NV}(x_1)$$

$$\left.\Big\}\,u_N(P_i,\,\lambda_i)\right. \qquad (4.7)$$

where we have made use of the Eqs. (2.16), (2.22), (3.4) and (3.12). The hard scattering amplitudes T_H are to be calculated from the Feynman diagrams shown in Fig. 2. In Eq. (4.7) no isospin changing diquark transitions (S–V and V–A_0) appear. For the 3-point contribution, i.e. for those contributions originating from the Feynman diagrams where the photon couples to the quark, this fact is an obvious dynamical property of the model: A gluon cannot mediate an isospin changing transition. For the 4-point contributions, on the other hand, where the photon couples to the diquarks, isospin changing transitions may occur in principle but we make the simplifying assumption that contributions from such transitions can be neglected. This assumption is justified by the fact that the 4-point contributions represent only corrections to the final results. Also in previous applications of the quark-diquark model [6, 9, 10] such contributions have been neglected. We mentioned in Sect. 3 that also qP and qA_1 configurations contribute to a N^* state which would lead to additional diquark-diquark transitions. Yet S–A_1 and V–P transitions change isospin and therefore do not contribute to N–N^* transitions or can be neglected. S–P transitions are strictly zero because of parity. Furthermore, we have excluded V–A_1 transitions. This assumption seems reasonable since, in an analogous calculation of the π–a_1 mesonic transition (see Appendix B), the V–A_1 transition can also be shown to vanish asymptotically, when the diquarks are resolved into quarks.

Thus, we are left with three different transitions, S–S, V–V and S–A_0, mediated either by a gluon or a photon. The corresponding vertices for S–S and V–V transitions, needed in the calculation of the hard scattering amplitudes have already been written down in [6, 9, 10]. They read

$$SgS : \qquad i\,g_S\,t^a(p_1 + p_2)_\mu \qquad (4.8)$$

$$VgV : \qquad -i\,g_S\,t^a\bigg[g_{\alpha\beta}(p_1 + p_2)_\mu - g_{\mu\alpha}[(1+\kappa_V)p_1 - \kappa_V p_2]_\beta$$

$$- g_{\mu\beta}[(1+\kappa_V)p_2 - \kappa_V p_1]_\alpha\bigg], \qquad (4.9)$$

where $g_S = \sqrt{4\pi\,\alpha_S}$ is the usual QCD coupling constant which appears here because diquarks are colour antitriplets. The kinematics used in (4.8) and (4.9) is defined in Fig. 3. κ_V is the anomalous magnetic moment of the vector diquark for which we use the value 1.39 [9]. $t^a = (\Lambda^a/2)$ are the Gell-Mann colour matrices. The form of the contact terms (γSgS, γVgV) required by gauge invariance is obvious. They can be found in Ref. [11]. The corresponding vertices with photons instead of gluons are obtained by replacing $g_S\,t^a$

15

by $-e_0\, e_D$ (e_D is the charge of the diquark D).

The new vertex which appears here for the process under investigation is the spin 0 - gluon (photon) - spin 1 diquark vertex $Sg(\gamma)A_0$. Its structure can be taken from the analogous π–a_1 transition current (B.1). There are two covariants in general and correspondingly two form factors. Their asymptotic behaviours are given by (B.10) and (B.11). Our numerical estimates of these form factors for the π–a_1 case provide the approximate relation $F_1 \approx Q^2 F_2$. Assuming that a similar relationship holds for the S–A_0 case we can write down, in the same spirit as we constructed the other diquark vertices, a simple gauge invariant vertex for S–A_0 transitions

$$ SgA_0 \; : \; i\, g_S\, \frac{t^a}{m_S}\, \left[q^2\, g_{\mu\alpha} \; - \; q_\mu q_\alpha \right] \, . \tag{4.10}$$

For on-shell diquarks this vertex reduces to (B.1) with $F_1 = Q^2 F_2$. As it will turn out the S–A_0 contributions only provide a small correction to the N–N^* transition form factors the ansatz (4.10) is sufficient. For the same reason we can also safely neglect S–A_0 4-point contributions.

In applications of the quark-diquark model Feynman diagrams are calculated with the rules for point-like particles. However, in order to take into account the composite nature of the diquarks phenomenological vertex functions have to be introduced. The 3-point functions, i.e. the ordinary diquark form factors, are parameterized such that asymptotically the diquark model evolves into the pure quark model of Brodsky and Lepage [1]. In view of this the diquark form factors are parameterized as

$$ \begin{aligned} F_S(Q^2) \; &= \; F_S^{(3)}(Q^2) \; = \; (1 \, + \, Q^2/Q_S^2)^{-1} && (4.11) \\ F_V(Q^2) \; &= \; F_V^{(3)}(Q^2) \; = \; (1 \, + \, Q^2/Q_V^2)^{-2} \, , && (4.12) \end{aligned}$$

and

$$ F_{SA}(Q^2) \; = \; F_{SA}^{(3)}(Q^2) \; = \; (1 \, + \, Q^2/Q_{SA}^2)^{-2} \, , \tag{4.13}$$

where the zero of the S–A_0 form factor at threshold is ignored; we are not interested in the small Q^2 region. In accordance with the correct asymptotic behaviour the 4-point functions are parameterized as

$$ F_S^{(4)}(Q^2) \; = \; a_S\, F_S(Q^2) \, ; \qquad F_V^{(4)}(Q^2) \; = \; a_V\, F_V(Q^2)\, (1 \, + \, Q^2/Q_V^2)^{-1} \, . \tag{4.14}$$

The required asymptotic behaviour of the diquark form factors can be calculated using the usual hard scattering picture along the same lines as for meson form factors ($\pi \to \pi$, $\rho \to \rho$, $\pi \to a_1$; for the latter see Appendix B). The first two form factors have been successfully applied in previous applications of the quark-diquark picture [6, 9, 11], where also the form factor parameters entering (4.11)-(4.14) have been determined. We take the values from Ref. [9]:

$$ Q_S^2 \; = \; 3.22\, \text{GeV}^2 \, ; \qquad Q_V^2 \; = \; 1.50\, \text{GeV}^2 \, . \tag{4.15}$$

For the sake of simplicity we set $Q_{SA}^2 = Q_V^2$. The physical picture underlying this assumption is that V- and A_0-diquarks both dissolve into quarks at a momentum scale which is smaller than that for S-diquarks. The constants a_S and a_V in the 4-point functions are strength

parameters, which take into account absorption due to diquark excitation and break-up. Again we take the values for a_S and a_V from Ref. [9]: $a_S = 0.15$; $a_V = 0.05$.

Having specified the rules for the phenomenological diquark vertices we are now in the position to calculate the hard scattering amplitudes and hence the current matrix elements (4.7). A comparison with the general covariant decomposition of the current matrix elements, Eq. (4.2), yields the helicity amplitudes of the form factors. Before presenting the results a remark concerning the K-dependence of the hard scattering amplitudes T_{HSS} and T_{HVV} is in order. Obviously, part of the K-dependence comes from the propagators. According to the kinematics defined in Fig. 2, the internal quark and gluon momenta that contribute to the 3-point functions given by the two Feynman diagrams read

$$
\begin{aligned}
q_G &= x_2\, P_i - y_2\, P_f + K \\
q_f &= P_f - x_2\, P_i \\
q_i &= P_i - y_2\, P_f + K \,.
\end{aligned}
\tag{4.16}
$$

To a very good approximation the denominators of the corresponding propagators are

$$
q_G^2 = -x_2 y_2 Q^2 + K^2 + \mathcal{O}(M_i^2, M_f^2)
\tag{4.17}
$$

$$
q_f^2 - m_q^2 = -x_2 Q^2 + \mathcal{O}(M_i^2, M_f^2)
\tag{4.18}
$$

$$
q_i^2 - m_q^2 = -y_2 Q^2 + K^2 + \mathcal{O}(M_i^2, M_f^2)\,.
\tag{4.19}
$$

For the Feynman diagrams contributing to the 4-point functions the internal momenta and propagator denominators are obtained from the above ones by the replacement $x_1 \leftrightarrow x_2$; $y_1 \leftrightarrow y_2$. We see that the denominators depend only on K^2. Hence, to the order we are working at, the denominator K-dependence can be neglected. Thus, the K-dependence of the hard scattering amplitude originates from the coupling of photons and/or gluons to the diquarks and from the quark (and diquark) propagator numerators related to the momentum q_i. From these terms we have to compute the derivative of the hard scattering amplitude with respect to K.

5 Results and comparison with data

For a qualitative discussion of our results we first write down the analytical expressions for the S–S contributions to the p–$S_{11}(1535)$ helicity form factors. These are obtained by working out (4.7) and then comparing the result with the general structure (4.2). One has

$$
\begin{aligned}
h_+^{S-S}(Q^2) &= -C_F\, \frac{8\pi}{3Q^2}\, \frac{f_{N^*S}}{M_f^2}\, f_{NS} \int_0^1 dx_1 dy_1\, \phi_{N^*S}(y_1)\, \frac{1}{y_1} \\
&\times \left[2\frac{\alpha_S(\tilde{Q}_{22}^2)}{x_2 y_2}\, F_S^{(3)}(\tilde{Q}_{22}^2) + \frac{\alpha_S(\tilde{Q}_{11}^2)}{x_1 y_1}\, F_S^{(4)}(\tilde{Q}_{11}^2 + \tilde{Q}_{22}^2) \right]\, \phi_{NS}(x_1)
\end{aligned}
\tag{5.1}
$$

$$
h_0^{S-S}(Q^2) = -C_F \frac{8\pi}{3Q^4} f_{N^*S} f_{NS} \int_0^1 dx_1 dy_1 \, \phi_{N^*S}(y_1)
$$
$$
\times \left[2\frac{\alpha_S(\tilde{Q}_{22}^2)}{x_2 y_2} F_S^{(3)}(\tilde{Q}_{22}^2)\left(1+\frac{x_1 M_i}{y_1 M_f}\right) + \frac{\alpha_S(\tilde{Q}_{11}^2)}{x_1^2 y_1^2} F_S^{(4)}(\tilde{Q}_{11}^2 + \tilde{Q}_{22}^2) \right.
$$
$$
\left. \times \frac{1}{2}\left(\left(1+\frac{x_1 M_i}{y_1 M_f}\right)(1+x_1 y_1 + x_2 y_2) - 4\right)\right] \phi_{NS}(x_1) , \qquad (5.2)
$$

where $\tilde{Q}_{ij}^2 \equiv x_i y_j Q^2$. Contrary to the case of elastic proton form factors the S diquarks contribute to the helicity changing N-N^* transitions (h_0^{S-S}). The reason for this is obvious: hadron helicity flip contributions can be mediated via the orbital angular momentum of the N^* without changing the quark helicity.

As discussed in Sect. 2 the $\ell = 0 \to \ell = 1$ transition current matrix element is given by the sum of two parts: The first one contains the $\ell = 1$ wave function to first order in $k_\perp$ and the derivative of the hard scattering amplitude with respect to $k_\perp$, and the second one contains the correction $\Delta\Gamma$ and the hard scattering amplitude to zeroth order in $k_\perp$. It is to be noted that only terms from the second part $\sim \Delta\Gamma$ contribute to h_+, whereas h_0 is determined by both parts. Furthermore, we note that $h_+^{S-S} \sim Q^{-4}$ and $h_0^{S-S} \sim Q^{-6}$ asymptotically (when use is made of Eqs. (4.11) and (4.14)), i.e. the transverse form factor dominates for large Q^2 as it should.

For the V–V contributions we obtain

$$
h_+^{V-V}(Q^2) = -C_F \frac{\pi}{3} f_{N^*V} f_{NV} \int_0^1 dx_1 dy_1 \, \phi_{N^*V}(y_1) \frac{\alpha_S(\tilde{Q}_{11}^2)}{x_1 y_1 M_f^2 m_V^2} F_S^{(4)}(\tilde{Q}_{11}^2 + \tilde{Q}_{22}^2)
$$
$$
\times \left[\frac{\kappa_V}{y_2}\left((1+x_1)\kappa_V + 2x_1\right)\left(3y_1 - 1 + (1+y_1)\frac{x_1 M_i}{y_1 M_f}\right) + \frac{y_1 M_f}{M_i}\kappa_V(\kappa_V - 1) \right.
$$
$$
\left. + (\kappa_V - 1)^2(2+3x_1) - 2\kappa_V + \frac{3-\kappa_V}{y_1}(x_2 + y_1 - 4x_1 y_1) \right] \phi_{NV}(x_1) \qquad (5.3)
$$

$$
h_0^{V-V}(Q^2) = -C_F \frac{\pi}{3} f_{N^*V} f_{NV} \int_0^1 dx_1 dy_1 \, \phi_{N^*V}(y_1) \frac{\alpha_S(\tilde{Q}_{11}^2)}{x_1 y_1 y_2 M_i M_f m_V^2} F_S^{(4)}(\tilde{Q}_{11}^2 + \tilde{Q}_{22}^2)
$$
$$
\times \left[\kappa_V(1+y_1)\left(\frac{M_i}{y_1 M_f}[2(1+x_1)(\kappa_V - 1) + 3x_1] - 1\right) + 2y_2(1+\kappa_V)^2 \right] \phi_{NV}(x_1). \qquad (5.4)
$$

Because of charge cancellation there are no 3-point V–V contributions to $p \to S_{11}(1535)$ like in the case of elastic proton form factors. Contrary to h_+^{S-S}, the leading order of h_+^{V-V} also receives contribution from terms involving derivatives of the hard scattering amplitude with respect to $k_\perp$. The major difference, however, is that asymptotically $h_+^{V-V} \sim Q^{-6}$, i.e. h_+^{V-V} is subdominant in its large Q^2 behaviour.

The contributions from S–A_0 diquark transitions read

$$
h_+^{S-A}(Q^2) = -C_F \frac{16\pi}{3\sqrt{3}} \frac{f_{N^*A_0} f_{NS}}{m_S} \int_0^1 dx_1 dy_1 \, \phi_{N^*A_0}(y_1) \frac{\alpha_S(\tilde{Q}_{22}^2)}{y_2 M_f^2} F_{SA}^{(3)}(\tilde{Q}_{22}^2) \phi_{NS}(x_1) \qquad (5.5)
$$

$$
\begin{aligned}
h_0^{S-A}(Q^2) &= C_F \frac{16\pi}{3\sqrt{3}} \frac{f_{N^*A_0} f_{NS}}{m_S} \int_0^1 dx_1 dy_1 \, \phi_{N^*A_0}(y_1) \\
&\times \frac{\alpha_S(\tilde{Q}_{22}^2)}{y_2 Q^2} F_{SA}^{(3)}(\tilde{Q}_{22}^2) \left(5y_2 - 1 - \frac{x_1 M_i}{M_f}\right) \phi_{NS}(x_1) \, .
\end{aligned}
\tag{5.6}
$$

Note that h_+^{S-A} and h_0^{S-A} behave as Q^{-6} asymptotically as in the $V-V$ case.

Throughout the running coupling constant is taken to be $\alpha_S(Q^2) = 12\,\pi \,/\, (25 \ln(Q^2/\Lambda_{\rm QCD}^2))$ with $\Lambda_{\rm QCD} = 200$ MeV. In order to avoid singularities for $Q^2 \to \Lambda_{\rm QCD}^2$ we cut off α_S at a value of 0.5. Keeping in mind that our model relies on perturbative methods with the implicit assumption that the running strong coupling constant $\alpha_S(Q^2)$ is small we do not believe our model to be applicable below $Q^2 \lesssim 4$ GeV^2.

In the expressions (5.1)–(5.6) there are a few free parameters, namely f_{N^*S} and f_{N^*V}, the "derivatives of the N^* wave function at the origin of the configuration space" and $f_{N^*A_0}$. The DAs and other parameters appearing in these expressions are taken from the study of the electromagnetic nucleon form factors [11]. We introduce a further simplification in this first explorative investigation of $\ell = 0 \to \ell = 1$ transitions by assuming $f_{N^*S} = f_{N^*V}$. Thus, altogether we are left with two parameters to be fitted to the experimental data.

In [26] a compilation is given of both exclusive and inclusive data on the transition $p \to S_{11}(1535)$ (see caption to Fig. 4 for details). The exclusive data, however, is limited to values of $Q^2 \leq 3$ GeV^2, whereas the inclusive data covers a range in Q^2 up to about 20 GeV^2. It is given in terms of the helicity amplitude $A_{1/2}$ which is proportional to the helicity conserving form factor h_+ [7, 27, 28]

$$
\begin{aligned}
A_{1/2} &= \sqrt{\frac{M_f}{M_f^2 - M_i^2}} \, \langle N^*, 1/2 \,|\, \epsilon_\mu(1) J^\mu \,|\, N, 1/2 \rangle \\
&= -e_0 \, M_f \sqrt{\frac{Q_+}{8 M_i \, (M_f^2 - M_i^2)}} \, h_+(Q^2) \, ,
\end{aligned}
\tag{5.7}
$$

where $Q_+ = (M_f + M_i)^2 - q^2$. For $Q^2 \geq 3$ GeV^2 the resonance cross section is dominated by the S_{11} around 1535 MeV invariant mass. Experimentally $A_{1/2}$ was determined from a resonance fit to the cross section where the assumption was made that contributions from helicity changing transitions to the cross section can be neglected (see Ref. [26] for details on the amplitude extraction procedure).

Frequently one uses also the magnetic transition form factor G_M which, in analogy to the Sachs form factors of the nucleon, is defined by

$$
G_M = -\frac{M_f}{2} \sqrt{\frac{Q_+}{Q_-}} \, h_+ \, .
\tag{5.8}
$$

In Fig. 4 we show the data on G_M [26, 29, 30] and our model results with $f_{N^*S} = f_{N^*V} = 0.08$ GeV^2 and $f_{N^*A_0} = 0.05$ GeV. For Q^2 larger than about 6 GeV^2 G_M behaves as Q^{-4} as is predicted by the hard scattering picture of Brodsky and Lepage [1]. Our values for G_M (or $A_{1/2}$) are much larger than those obtained by Carlson and Poor [5] who used the

19

Brodsky-Lepage model and the QCD sum rule constrained Chernyak-Zhitnitsky DA [15]. As already mentioned in the introduction Carlson and Poor's ansatz does not treat the orbital angular momentum of the S_{11} properly. For comparison we also display the results obtained by Konen and Weber [7] in Fig. 4. Because the Konen-Weber model is a relativistically generalized constituent quark model which does not account for hard processes, its validity is limited to the low Q^2 regime ($\lesssim 3$ GeV^2).

In Fig. 5 we plot the helicity form factors h_+ and h_0 (multiplied by Q^4) and their decomposition into the contributions from the various diquark transitions (Eqs.(5.1)-(5.6)). We observe that the dominant contribution to h_+ is provided by the S–S contribution. The V–V and S–A_0 transitions only supply very small corrections to h_+ which could even be ignored without degrading the quality of the fit. This result justifies our crude treatment of the S–A_0 transitions which represents a new piece in the diquark model. Obviously, since the S–A_0 contribution is so small, the parameter $f_{N^*A_0}$ is not well constrained by the data. For the other helicity form factor h_0 the situation is quite different. The contributions from the V–V and the S–A_0 transitions are not negligible as compared to those from the S–S transitions. Data on that form factor would pin down the magnitude of the S–A_0 contributions. There are a few data points on h_0 in the 2–3 GeV^2 region [31]. Fig. 5 shows that the trend of our results is in fair agreement with the data on h_0.

6 Summary and conclusions

In this paper we have proposed a method to extend the hard scattering picture of Brodsky and Lepage [1] to hadrons with nonzero orbital angular momentum where one can no longer neglect the internal transverse momentum. We have applied this formalism to the N–N^* transition in the quark-diquark picture and to the mesonic transition $\pi \rightarrow a_1$. The generalization of our approach to baryon reactions in the pure quark picture is straightforward and will be performed in a later paper. We emphasize that our formalism as well as the original Brodsky-Lepage approach is only valid at large momentum transfer. In the few GeV region, however, radiative corrections [2] have to be taken into account as well. In the present explorative study we have ignored this complication.

Concerning possible quark-diquark configurations inside the N^* the orbital angular momentum can either be between quark and diquark or reside inside the diquark. In the former case one encounters the well-known S- and V-diquarks, whereas the latter case requires the introduction of new diquarks with new couplings. However, only three configurations contribute to the N–N^* transition, namely the diquark transitions $S \rightarrow S$, $V \rightarrow V$ and the new transition $S \rightarrow A_0$. In the numerical analysis we found that the contributions from S–S diquark transitions provide the dominant part of the non-flip helicity form factor h_+, whereas the behaviour of the helicity flip form factor h_0 is mainly governed by V–V and S–A_0 transitions.

It is important to note that the phenomenology of this treatment is quite different from

that of Refs. [5, 6], where the N^* was treated uncorrectly as a nucleon with opposite parity. Although our model of the N^* involves some unknown parameters we believe that our approach constitutes a step forward towards an understanding of hadrons with nonzero orbital angular momentum.

Acknowledgements

Appendix A: Derivation of the spin wave functions for the $N(1/2^+)$ and the $N^*(1/2^-)$ in the ls coupling scheme

One way of constructing covariant spin wave functions is to make use of the observation [14] that, in the zero binding energy approximation, an equal-t hadron state in the constituent center of mass frame ($\sum \vec{p}_j = 0$) equals the light cone state at rest. Consequently, one can use the standard ls coupling scheme to couple quarks and diquarks in the baryon case, or quarks and antiquarks in the meson case to form a state of given spin and parity. On boosting the result to a frame with arbitrary hadron momentum P^μ one can easily read off the covariant spin wave function. As was mentioned in Sect. 2 one needs K^α-corrections to the spin wave functions for $\ell \neq 0$ hadrons. As we will see below the approach presented in this appendix also provides a scheme for calculating such corrections which admittedly is model-dependent.

To be specific we will describe the construction of covariant spin wave functions in the ls coupling scheme for spin $1/2$ baryons made of quarks and diquarks of type j with spin s_j and with a relative orbital angular momentum ℓ between quark and diquark. The generalization to other cases is straightforward. The center of mass momenta are denoted by

$$\hat{P}^\mu = (M, \vec{0})\,; \qquad \hat{p}_1^\mu = (m_1, \vec{k})\,; \qquad \hat{p}_2^\mu = (m_2, -\vec{k})\,, \qquad (A.1)$$

and, according to the discussion in Sect. 2, we have the approximate relations $m_i = x_i M + \mathcal{O}(k^2/M)$, $i = 1, 2$. The baryon is represented by the equal-t spinor $u_B(\hat{P}, \mu)_t$ and its constituents, the quark and the diquark j, by the spinors $u(\hat{p}_1, \mu_1)_t$ and by $\hat{\epsilon}_j(\hat{p}_2, \mu_2)$ which is 1 for a scalar diquark and a Lorentz vector for the case $s_j = 1$. Note that μ, μ_1 and μ_2 denote spin components. The ls coupling scheme leads to the following ansatz for the spin wave

function

$$\hat{\Gamma}_{j\ell}\, u_B(\hat{P},\mu)_t \;=\; \sum_{\mu_1\mu_2\mu_\ell} |\,\vec{k}\,|^{\,\ell}\,\sqrt{4\pi}\, Y_{\ell\mu_\ell}(\vec{k}/\,|\vec{k}|\,)$$

$$\times \left(\begin{array}{cc|c} 1/2 & s_j & s \\ \mu_1 & \mu_2 & \mu_s \end{array} \right) \left(\begin{array}{cc|c} s & \ell & 1/2 \\ \mu_s & \mu_\ell & \mu \end{array} \right) u(\hat{p}_1,\mu_1)_t\,\hat{\epsilon}_j(\hat{p}_2,\mu_2)\,. \qquad \text{(A.2)}$$

Possible Lorentz indices of the baryon spin wave function and of $\hat{\epsilon}_j$ are omitted for the present discussion. The quark spinor is related to the baryon spinor by

$$u(\hat{p}_1,\mu_1)_t \;=\; \frac{1}{2x_1 M}\,(x_1\,\hat{\slashed{P}} + \ \hat{\slashed{K}} + x_1 M)\,u_B(\hat{P},\mu_1)_t\,, \qquad \text{(A.3)}$$

where the 4-vector $\hat{K}^\mu = (0,\vec{k})$ has been introduced. Up to corrections of order k^2/M^2 the polarization vector of a $s_j\!=\!1$ diquark can be expressed as

$$\hat{\epsilon}_j{}^\nu(\hat{p}_2,\mu_2) \;=\; \left(g^\nu{}_\rho\, +\, \frac{1}{x_2 M^2}\,\hat{P}^\nu \hat{K}_\rho\right)\hat{\epsilon}_j{}^\rho(\hat{P},\mu_2)\,, \qquad \text{(A.4)}$$

where $\hat{\epsilon}_j{}^\rho(\hat{P},\mu_2)$ describes the polarization state of a vector diquark at rest. Thus, inserting (A.3) and (A.4) in (A.2), one can easily find the rest frame spin wave functions including, for $\ell\!=\!1$, the corrections proportional to K^α. Boosting to a frame with baryon momentum $P^\mu\, =\, (E,\vec{P})$ one arrives at the desired spin wave functions written down in Sect. 3 (and for the π and a_1 meson in Appendix B). They include the corrections $\Delta\Gamma$ obtained from the expansion of the quark spinor and the polarisation vector around the direction of the hadron momentum. Since this has been carried out for free quark spinors respective polarisation vectors the result for $\Delta\Gamma$ is model dependent in contrast to the leading terms of the spin wave functions which only depend on the hadronic quantum numbers and the type of the constituents.

Appendix B: The $\pi-a_1$ form factors

In this Appendix we present a calculation of the $\pi-a_1$ form factors at large momentum transfer. The reason for doing this is twofold. On the one hand we want to demonstrate that our approach is also applicable to mesonic transitions and, on the other hand, we can read off the asymptotic behaviour of the $S-A_0$ diquark form factors from the results. We need to know the large Q^2 power dependence for the parameterization of the diquark form factors in order to guarantee that asymptotically the pure quark model emerges from the quark-diquark picture.

We use analogous notations and the same frame as for the calculation of the $N-N^*$ transition form factors (see Sect. 2). We covariantly decompose the current matrix element as follows

$$\langle\, a_1;\, \vec{P}_f,\, \lambda_f \mid J^\mu \mid \pi;\, \vec{P}_i\,\rangle =$$
$$-i\,e_0\,\Big[\, F_1(Q^2)\,(q\!\cdot\! P_f\, g^{\mu\nu}\, +\, P_f^\mu P_i^\nu)\, +\, F_2(Q^2)\,(q\!\cdot\! P_f\, P_i^\mu\, -\, q\!\cdot\! P_i\, P_f^\mu)\, P_i^\nu\Big]\,\epsilon_\nu\,. \qquad \text{(B.1)}$$

The covariants are manifestly gauge invariant. Using again the opposite momentum brick wall frame one concludes that F_1 is the form factor for transverse transitions and, for $Q^2 \to \infty$, F_2 is the form factor for longitudinal transitions.

According to our statements made in Sect. 2 we write the mesonic states as [19]

$$| \pi; \vec{P}_i \rangle = \int dx_1 \, f_\pi \, \phi_\pi(x_1) \, \Gamma_\pi \tag{B.2}$$

$$| a_1; \vec{P}_f, \lambda_f \rangle = \int dy_1 \, f_{a_1} \, \phi_{a_1}(y_1) \, (\Gamma_{a_1}^\alpha + \Delta\Gamma_{a_1}^{\alpha\beta}), \tag{B.3}$$

where

$$\Gamma_\pi = \frac{1}{\sqrt{2}} (\not{P}_i + M_i) \gamma_5 \tag{B.4}$$

$$\Gamma_{a_1}^\alpha = \frac{1}{2} (\not{P}_f + M_f) [\not{\epsilon}(\lambda_f), \gamma^\alpha] \gamma_5, \tag{B.5}$$

$$\tag{B.6}$$

and

$$\Delta\Gamma_{a_1}^{\alpha\beta} = g(y_1) \gamma^\beta [\not{\epsilon}(\lambda_f), \gamma^\alpha] \gamma_5. \tag{B.7}$$

The spin wave function Γ_{a_1} (Eq. (B.5)) has been given before in [20, 24, 32]. The correction term $\Delta\Gamma_{a_1}^{\alpha\beta}$ takes into account that quark and diquark are not collinear with the a_1 meson. The model dependence of this term is absorbed into the function $g = g(y_1)$. Using the same approach as for the baryon spin wave functions (see appendix A), we find

$$g(y_1) = \frac{1}{4y_1 y_2}. \tag{B.8}$$

We have used the expansion of the free quark spinors around the direction of the a_1 momentum. To lowest order in α_S the hard scattering amplitude is given by (see Eq. (2.22))

$$\langle a_1; \vec{P}_f, \lambda_f \,|\, J^\mu \,|\, \pi; \vec{P}_i \rangle = -ie_0 \int dx_1 \, dy_1 \, f_{a_1} \, \phi_{a_1}^*(y_1) \frac{4\pi \, \alpha_S(q_G^2) \, C_F}{q_G^2} \, I_{\alpha\beta}/2$$

$$\times \; \mathrm{Tr} \left\{ \left[\bar{\Gamma}_{a_1}^\alpha \frac{\gamma^\mu \gamma^\beta \gamma^\nu}{q_i^2 - m_q^2} + \Delta\bar{\Gamma}_{a_1}^{\alpha\beta} \left(\frac{\gamma^\mu \not{q}_i \gamma^\nu}{q_i^2 - m_q^2} + \frac{\gamma^\nu \not{q}_f \gamma^\mu}{q_f^2 - m_q^2} \right) \right] \Gamma_\pi \gamma_\nu \right\} f_\pi \, \phi_\pi(x_1). \tag{B.9}$$

In writing Eq. (B.9) we have made use of the fact that the meson wave functions are symmetric under the replacement $x_1 \leftrightarrow x_2$ ($y_1 \leftrightarrow y_2$) due to time reversal invariance. The propagators are defined in Eq. (4.19). The trace can easily be worked out. At large Q^2 comparison with (B.1) results in the following expressions for the two form factors

$$F_1(Q^2) = \frac{4\pi \, C_F \, f_\pi \, f_{a_1}}{Q^4} \int dx_1 \, dy_1 \, \phi_{a_1}^*(y_1) \, \phi_\pi(x_1) \, \alpha_S(q_G^2) \frac{4}{x_2^2 y_2^2} \, (x_2 + 2g(1+x_2)y_2) \tag{B.10}$$

$$F_2(Q^2) = \frac{4\pi \, C_F \, f_\pi \, f_{a_1}}{Q^6} \int dx_1 \, dy_1 \, \phi_{a_1}^*(y_1) \, \phi_\pi(x_1) \, \alpha_S(q_G^2) \frac{64g}{x_2 y_2}, \tag{B.11}$$

where we have neglected the meson masses for convenience. The additional Q^{-2} factor in (B.11) is a consequence of the definitions of the covariants in (B.1). F_2 is not suppressed asymptotically in observables relative to F_1. For the model function (B.8) the end-point regions, $x_i(y_i) \to 0$, $i = 1, 2$, are rather dangerous. With distribution amplitudes of the type (3.7) and (3.8) which are damped exponentially in the end-point regions there is, however, no singularity (except of the familiar difficulty with α_S). When Sudakov corrections are taken into account in the manner proposed by Sterman et al. [2] the end-point regions are sufficiently strongly damped for any DA in use. Also the problem with α_S disappears in this case.

With regard to the Sudakov corrections, which are not taken into account in the present explorative study of $\ell = 0 \to \ell = 1$ transitions, we evaluate the π–a_1 transition form factors only for DAs strongly damped in the end-point regions. We consider results obtained with other DAs as unreliable overestimates of the form factors. The following pion DAs are used in the evaluation of the form factors: The "non-relativistic" DA $\sim \delta(x_1 - 1/2)$, the asymptotic form $\sim x_1 x_2$ and the CZ form $\sim x_1 x_2 (x_1 - x_2)^2$; the latter two are multiplied with an exponential $\exp(-\beta^2 m_q^2 / x_1 x_2)$ [33]. For details and the values of the oscillator parameter β and of the quark mass m_q, see Ref. [17]. The constant f_π is related to the pion decay constant by 133 MeV$/2\sqrt{6} = 27.15$ MeV. Not much is known about the DA of the a_1. Ref. [32] contains an attempt of estimating it. We will, however, not use this result but rather assume that the DA of the a_1 equals the pion's DA. The constant f_{a_1}, which is the derivative of the configuration space wave function at the origin, can be determined from the decay width $\Gamma(\tau \to a_1 \nu_\tau)$ which equals $\Gamma(\tau \to 3\pi\nu_\tau) = (1.47 \pm 0.13) \cdot 10^{-10}$ MeV to a very good approximation [34]. The usual decay constant is defined by $\langle 0 | J_\mu^W | a_1 \rangle = f_{a_1}^{\text{dec}} p_\mu$ and is related to the constant f_{a_1} by $f_{a_1} = f_{a_1}^{\text{dec}}/4\sqrt{3}$. The experimental decay width leads to $f_{a_1} = 0.025 \pm 0.002 \, \text{GeV}^2$. Our numerical results for the two π–a_1 form factors are shown in Fig. 6. It can be seen that the various choices in the DAs lead to quite different results for the form factors. The comparatively higher values obtained from the CZ DAs are due to their stronger concentration in the end-point regions. The ratio $Q^2 F_1/F_2$, however, can be found to be equal to a constant of order 1. We emphasize that we have calculated the large Q^2 behaviour of a form factor, namely F_1, which belongs to a helicity flip current matrix element. Hadronic helicity is not conserved in this case. The physical reason is obvious: The hadronic helicity is changed via the orbital angular momentum; as in the $\ell = 0$ case the quark helicities remain unchanged in the interaction with photons or gluons (quark masses are neglected). We mention that the π–a_1 form factors have been investigated in Ref. [35] at small, time-like momentum transfer.

References

[1] G. P. Lepage and S. J. Brodsky, Phys. Rev. D22 (1980) 2157

[2] H. Li and G. Sterman, Nucl. Phys. B381 (1992) 129; H. Li, Phys. Rev. D48 (1993) 4243

[3] J.H. Kühn, J. Kaplan and E.G.O. Safiani, Nucl. Phys. B157 (1979) 125

[4] B. Gueberina, J.H. Kühn, R.D. Peccei and R. Rückl, Nucl. Phys. B174 (1980) 317

[5] C.E. Carlson and J.L. Poor, Phys. Rev. D38 (1988) 2758

[6] P. Kroll, M. Schürmann and W. Schweiger, Z. Phys. A342 (1992) 429

[7] W. Konen and H.J. Weber, Phys. Rev. D41 (1990) 2201

[8] H.J. Weber, Ann. Phys. (N.Y) 177 (1987) 38; Z. Dziembowski and H.J. Weber, Phys. Rev. D37 (1988) 1289

[9] P. Kroll, Th. Pilsner, M. Schürmann and W. Schweiger, Phys. Lett. B316 (1993) 546; R. Jakob, P. Kroll, M. Schürmann and W. Schweiger, Z. Phys. A347 (1993) 109

[10] P. Kroll, M. Schürmann and W. Schweiger, Proceedings of the Workshop on Quark Cluster Dynamics, Bad Honnef (1992), eds. K. Goeke, P. Kroll and H. Petry, Springer Lecture Notes in Physics, Vol. 417, Heidelberg (1993)

[11] P. Kroll, M. Schürmann and W. Schweiger, Z. Phys. A338 (1991) 339

[12] P.A.M. Dirac, Rev. Mod. Phys. 21 (1949) 392

[13] H. Leutwyler and J. Stern, Ann. Phys. (N.Y.) 112 (1978) 94

[14] Z. Dziembowski, Phys. Rev. D37 (1988) 768

[15] V.L. Chernyak, A.A. Oglobin and I.R. Zhitnitsky, Z. Phys. C42 (1989) 583

[16] A. V. Radyushkin, Nucl. Phys. A532 (1991) 141c

[17] R. Jakob and P. Kroll, Phys. Lett. B315 (1993) 463, B319 (1993) 545(E)

[18] G.T. Bodwin, E. Braaten and G.P. Lepage, Phys. Rev. D46 (1992) 1914

[19] F. Hussain, J.G. Körner and G. Thompson, Ann. Phys. (N.Y.) 206 (1991) 334

[20] S. Balk, J.G. Körner, G.Thompson and F. Hussain, Z. Phys. C59 (1993) 283

[21] R. Barbieri, R. Gatto and R. Kögerler, Phys. Lett. 60B (1976) 185

[22] N. Isgur and C.H. Llewellyn-Smith, Nucl. Phys. B317 (1989) 526

[23] J.J. Aubert et al., Phys. Lett. B95 (1980) 306; A. Schlagböhmer, Ph.D. thesis, Freiburg (1986)

[24] "Hadrons of Arbitrary Spin and Heavy Quark Symmetry", F. Hussain, J.G. Körner and G. Thompson, preprint MZ-TH/93-23, University of Mainz

[25] R. C. E. Devenish, T.S. Eisenschitz and J.G. Körner, Phys. Rev. D14 (1976) 3063

[26] P. Stoler, Phys. Rev. Lett. 66 (1991) 1003; Phys. Rev. D44 (1991) 73; Phys. Rep. 226 (1993) 103

[27] D. H. Lyth, Electromagnetic interactions of hadrons; A. Donnachie and G. Shaw (eds.), vol. I, New York, Plenum Press (1978)

[28] M. Warns, W. Pfeil, H. Rollnik, Phys. Rev. D42 (1990) 2215

[29] F. W. Brasse et al., Nucl. Phys. B110 (1976) 410

[30] R. Haidan, PhD thesis, Univ. of Hamburg, unpublished, DESY internal report DESY F21-79/03 (1979)

[31] F. Foster and G. Hughes, Rep. Prog. Phys. 46 (1983) 1445; B. Boden and G. Krösen, Proceedings of the Continuous Electron Beam Accelerator Facility (CEBAF) Summer Workshop, Newport News Virginia, 1986, eds. V. Burkert et al. (CEBAF, Newport News, 1987), Vol. II.

[32] C.-R. Ji, P.L. Chung and S.R. Cotanch, Phys. Rev. D45 (1992) 4214

[33] G.P. Lepage, S.J. Brodsky, T. Huang and P.B. Mackenzie, Banff Summer Institute, Particles and Fields 2, eds. A.Z. Capri and A.N. Kamal (1993) p. 83

[34] H. Albrecht et al., ARGUS collaboration, Z. Phys. C58 (1993) 61

[35] I.G. Aznauryan and K.A. Oganessyan, Phys. Lett. B249 (1993) 309

Figure captions

Fig. 1: The N–N^* transition vertex.

Fig. 2a: Feynman diagrams contributing to the N–N^* transition form factors to leading order. The upper lines represent the quarks, the lower ones the diquarks. The 4-point function is drawn schematically as a blob and is explained in b) and c).

Fig. 2b: Diagrammatical decomposition of the $pSgS$ and the $pVgV$ 4-point function.

Fig. 3a: Diquark-photon/gluon-diquark 3-point vertices. μ, α and β denote Lorentz indices; a the colour of the gluon.

Fig. 3b: Photon-diquark-gluon-diquark 4-point vertices. q and k denote the momenta of the photon and the gluon, respectively.

Fig. 4: The magnetic transition form factor G_M (Eq. (5.8)) vs. Q^2. The solid line represents the prediction of the diquark model; the dashed line is the result of Konen and Weber [7] and the dotted line that of Carlson and Poor [5]. Data are taken from [26] ($\triangledown$ old, $\circ$ new inclusive SLAC data), [29] ($\bullet$) and [30] ($\diamond$).

Fig. 5: Helicity form factors h_+ (top) and h_0 (bottom) and their decomposition into contributions from the various diquark transitions (Eqs. (5.1)-(5.6)): Dashed line are S–S transitions, dotted line are V–V and dashed-dotted line are S–A_0 transitions. The solid line represents the full result.

Fig. 6: The π–a_1 transition form factors F_1 and F_2 (Eqs. (B.10) and (B.11)) vs. Q^2. The lines represent our results for the "non-relativistic" (solid), the asymptotic (dashed) and the CZ (dotted) DA, assuming that the π DA and a_1 DA are equal. In all cases the oscillator parameter in the DA was taken such as to yield a rms transverse momentum of 250 MeV assuming the $k_\perp$ dependences of the complete wave function to be of the form (3.9) for the π and (3.13) for a_1.

This figure "fig1-1.png" is available in "png" format from:

http://arXiv.org/ps/hep-ph/9403319v2

This figure "fig2-1.png" is available in "png" format from:
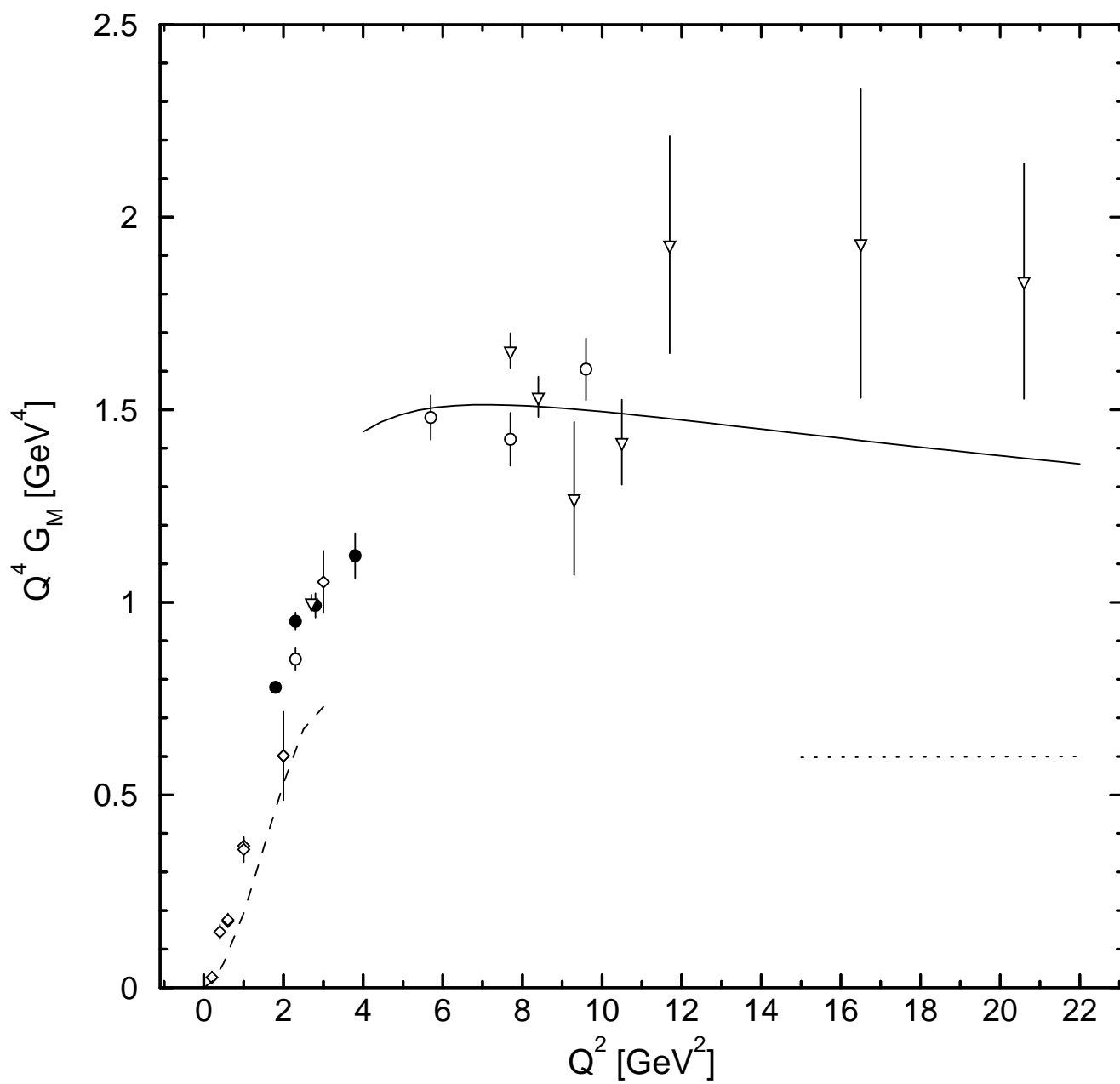
http://arXiv.org/ps/hep-ph/9403319v2
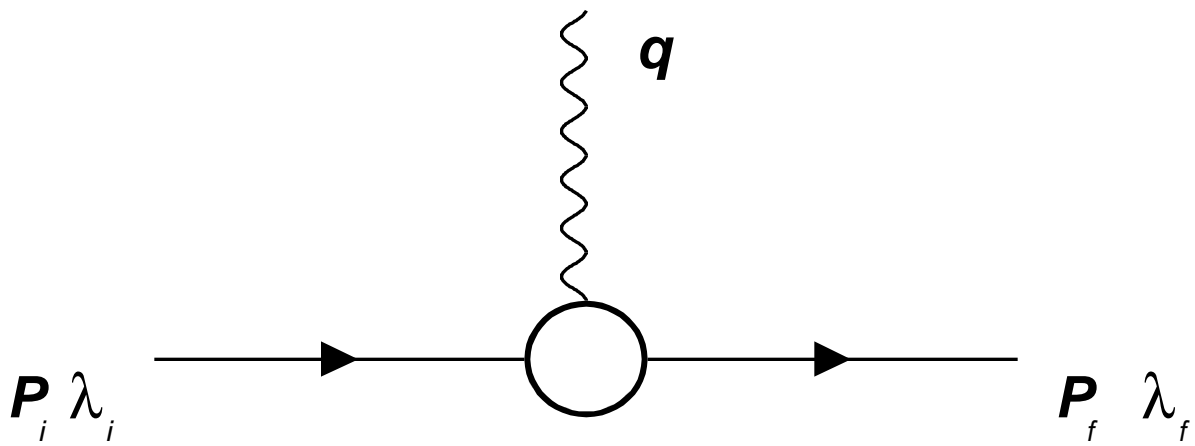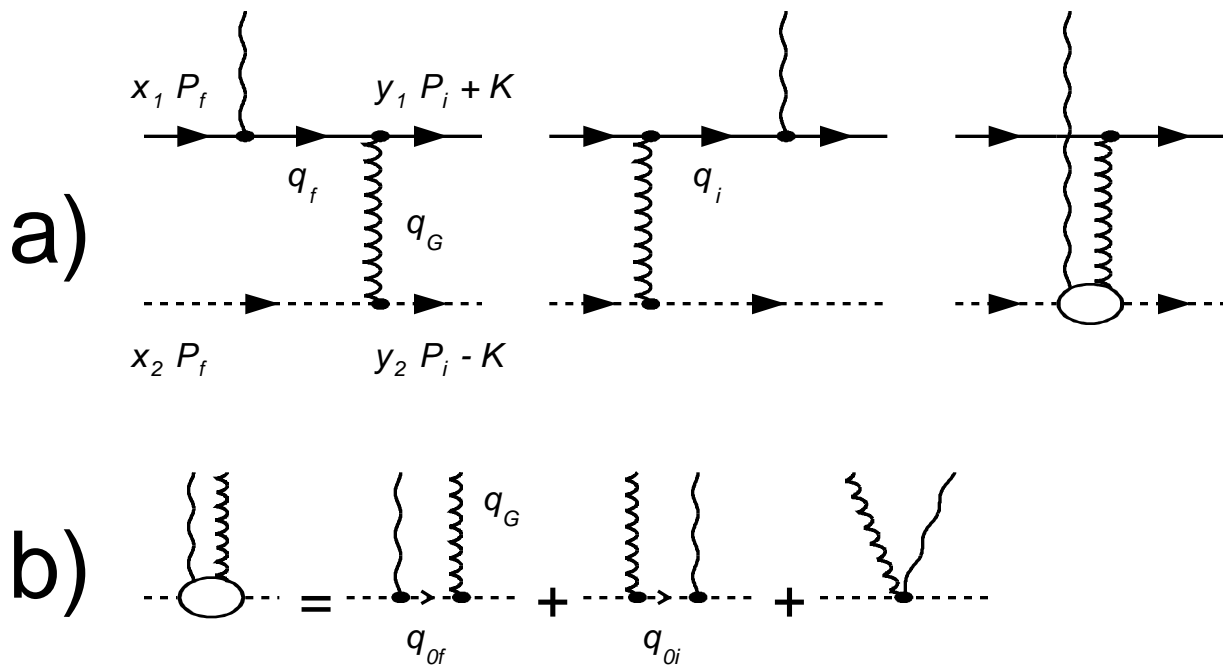
Figure 1

Figure 2

This figure "fig1-2.png" is available in "png" format from:

http://arXiv.org/ps/hep-ph/9403319v2

q (a, μ)

p_1 (α) p_2 (β)

(a)

q (μ) k (a, ν)

p_1 (α) p_2 (β)

(b)
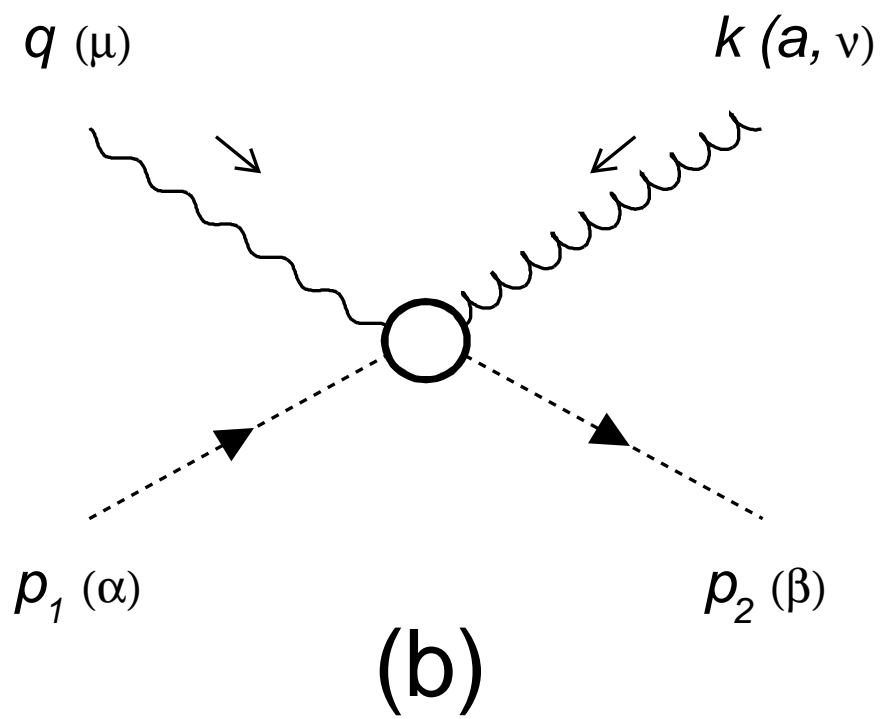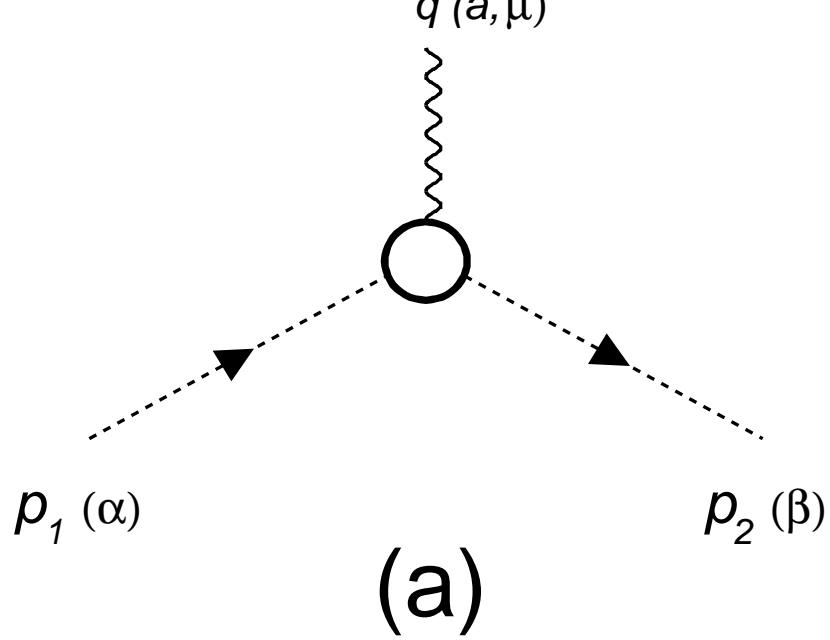
Figure 3

This figure "fig1-3.png" is available in "png" format from:

http://arXiv.org/ps/hep-ph/9403319v2

Figure 4

This figure "fig1-4.png" is available in "png" format from:

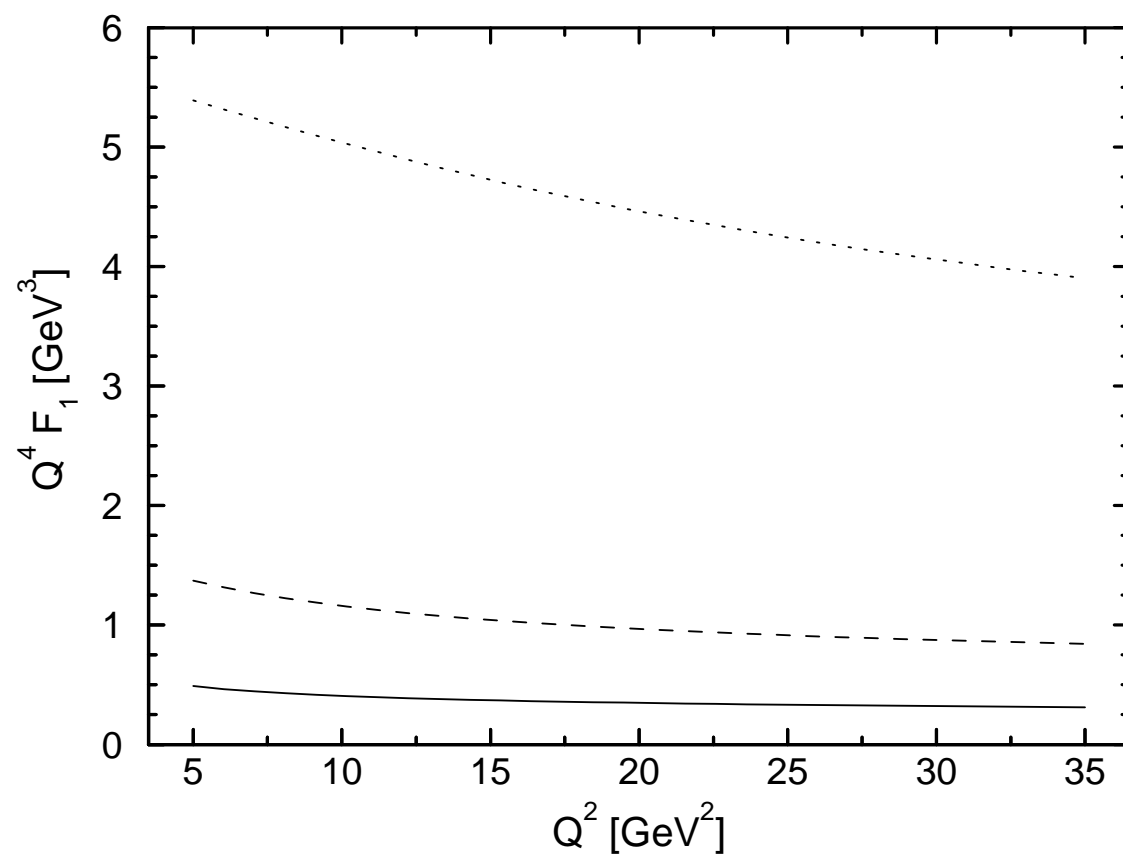http://arXiv.org/ps/hep-ph/9403319v2

Figure 5

Figure 6